



Sriraam  
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Massaoudi



## Lifted Message Passing



NeSy 2010 @ AAAI 2010, Atlanta, USA July 11, 2010

## Rorschach Test



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## Etzioni's Rorschach Test for Computer Scientists



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## Moore's Law?



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## Storage Capacity?



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## Number of Facebook Users?



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## Number of Scientific Publications?



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## Number of Web Pages?



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## Number of Actions?



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## Computing 2020: Science in an Exponential World



"The amount of scientific data is doubling every year"

[Szalay, Gray; *Nature* 440, 413-414 (23 March 2006) ]

How to deal with millions of images ?

How to deal with millions of inter-related research papers ?

How to accumulate general knowledge automatically from the Web ?

How to deal with billions of shared users' perceptions stored at massive scale ?

How do realize the vision of social search?



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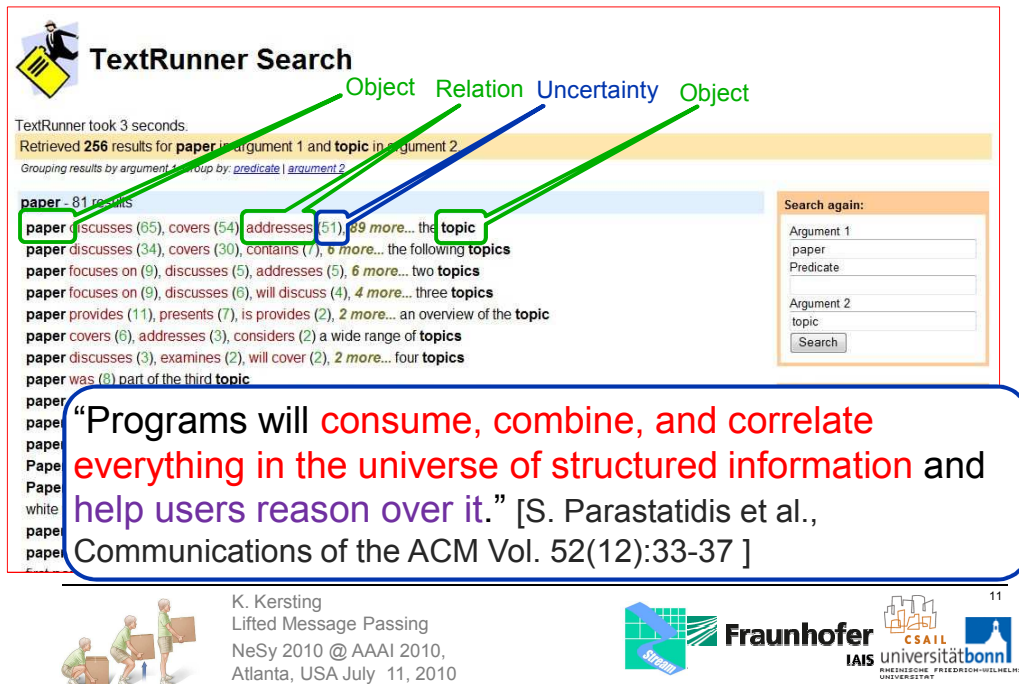
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**TextRunner Search**

TextRunner took 3 seconds.  
Retrieved 256 results for **paper** in argument 1 and **topic** in argument 2  
Grouping results by argument 1: group by: [predicate](#) | [argument 2](#)

**paper** - 81 results

**paper** discusses (65), covers (54), **addresses (51) 89 more... the topic**

**paper** discusses (34), covers (30), contains (7), **6 more... the following topics**

**paper** focuses on (9), discusses (5), addresses (5), **6 more... two topics**

**paper** focuses on (9), discusses (6), will discuss (4), **4 more... three topics**

**paper** provides (11), presents (7), is provides (2), **2 more... an overview of the topic**

**paper** covers (6), addresses (3), considers (2) a wide range of **topics**

**paper** discusses (3), examines (2), will cover (2), **2 more... four topics**

**paper** was (8) part of the **third topic**

**paper**

**paper**

**paper**

**Pape**

**Pape**

**white**

**paper**

**paper**

Search again:

Argument 1  
paper  
Predicate

Argument 2  
topic  
Search

**“Programs will consume, combine, and correlate everything in the universe of structured information and help users reason over it.” [S. Parastatidis et al., Communications of the ACM Vol. 52(12):33-37 ]**

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## Artificial Intelligence in an Exponential World

AI = **Structured** (**Data** + **Model** + **Reasoning**)

- Real world is structured in terms of objects and relations
    - Relational knowledge can reveal additional correlations between variables of interest . Abstraction allows one to compactly model general knowledge and to move to complex inference
- [Fergus et al. 30(11) 2008; Halevy et al., IEEE Intelligent Systems, 24 2009]
- Most effort has gone into the modeling part
  - How much can the data itself help us to solve a problem?



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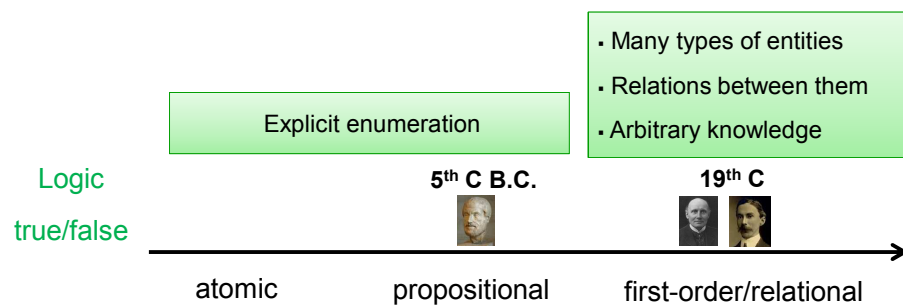
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**(First-order) Logic handles Complexity**

E.g., rules of chess (which is a tiny problem):

1 page in first-order logic,

~100000 pages in propositional logic,

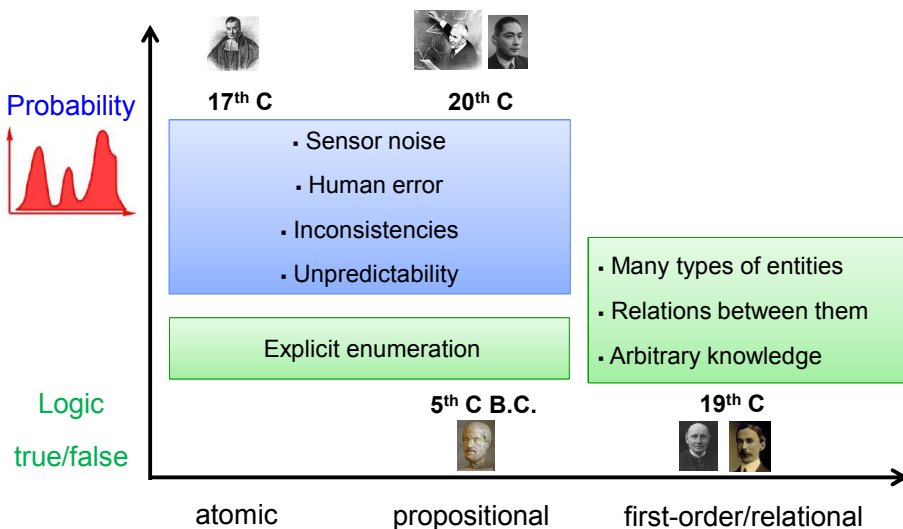
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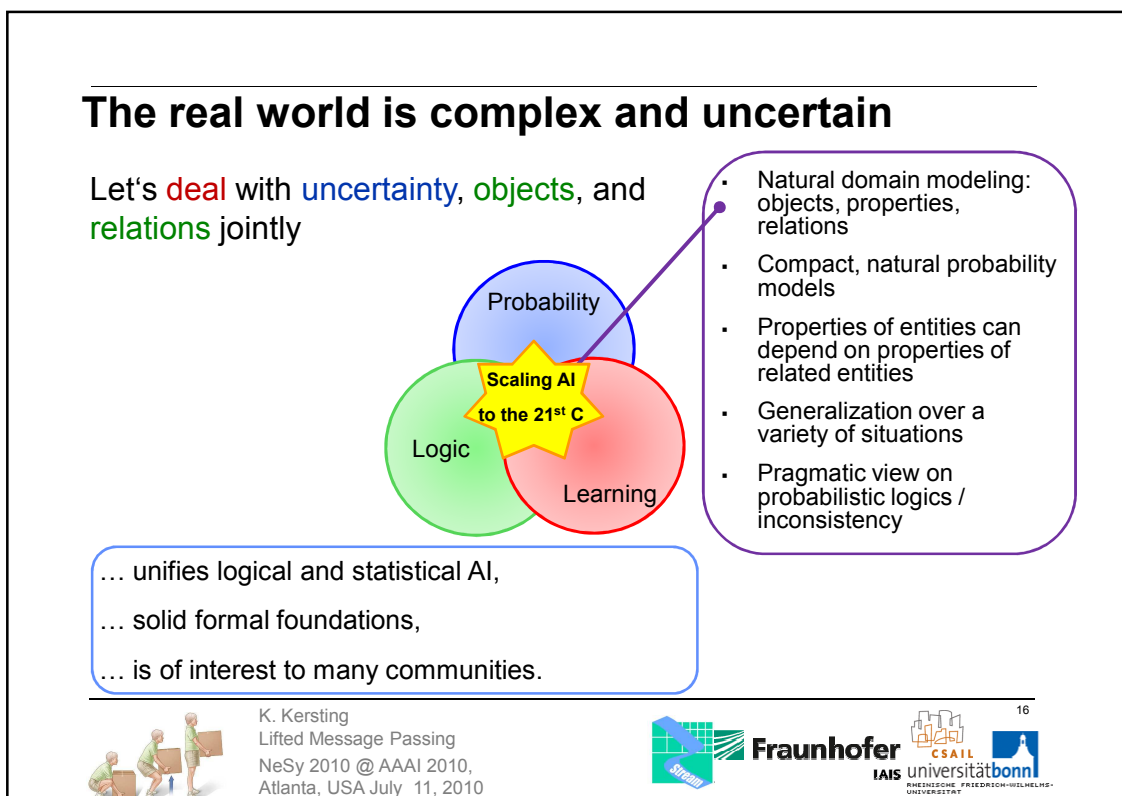
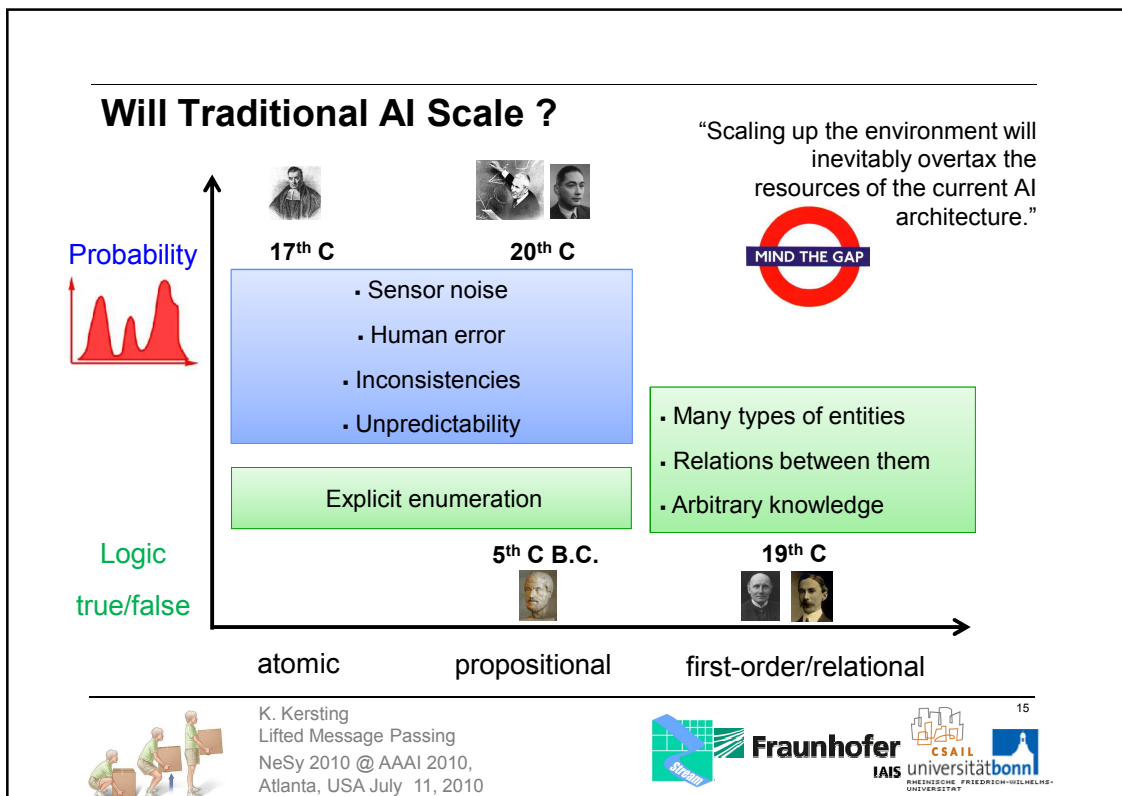
## Probability handles Uncertainty



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## The real world is complex and uncertain



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AAAI Workshop  
Summer 2000  
[Pointers](#)  
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### Learning Statistical Models from Relational Data

#### AAAI 2000 Workshop


The [AAAI 2000](#) Workshop on Learning Statistical Models from Relational Data was held on July 31, 2000 in Austin, Texas. The Workshop brought together researchers from diverse research areas, including machine learning, inductive logic programming, statistics, and databases. The workshop included nine paper presentations and two invited talks. The workshop closed with a roundtable discussion of potential application domains. Additional details on the [schedule](#) are given below. The collected papers from the workshop are available as a [AAAI Press technical report](#).



The bulk of the research presented at the workshop shared a common motivation: to uncover patterns and make predictions from structured data. However, there are multiple paths toward the common goal of statistical relational learning (SRL). One path begins with machine learning and statistical methods for "flat" or attribute-value representations, and expands these approaches to incorporate relational structure, techniques -- independent and identically distributed instances: structured data may introduce important statistical errors. A nonprobabilistic domains, especially inductive logic programming research area and several new languages and learning algorithms.

There was general consensus that a longer workshop should be held in the future to allow for a more in-depth synthesis of the many different approaches and applications.

**Workshop Co-chairs**

[Lise Getoor](#) Stanford University [getoor@cs.stanford.edu](mailto:getoor@cs.stanford.edu)  
[David Jensen](#) University of Massachusetts Amherst [jensen@cs.umass.edu](mailto:jensen@cs.umass.edu)





## Today, we can ...

- ... learn probabilistic relational models automatically from millions of inter-related objects
- ... generate optimal plans and learn to act optimally in uncertain environments involving millions of objects and relations among them
- ... perform lifted probabilistic inference avoiding explicit state enumeration by manipulating first-order state representations directly
- ... exploit shared factors to speed up message-passing algorithms for relational inference but also for classical propositional inference such as solving SAT problems



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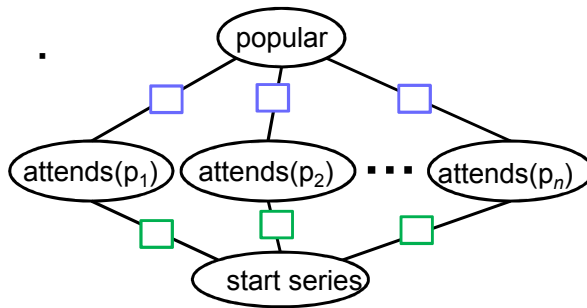
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## Lifted Inference

[Pfeffer et al. 1999; Poole 2003; de Salvo Braz et al. 2005]

- Example: Inviting  $n$  people to a workshop

### Factor graph



### Parfactors

$$\forall X. \phi_1(\text{popular}, \text{attends}(X))$$

$$\forall X. \phi_2(\text{attends}(X), \text{series})$$

- Lifted inference **exploits symmetries** revealed by relational model



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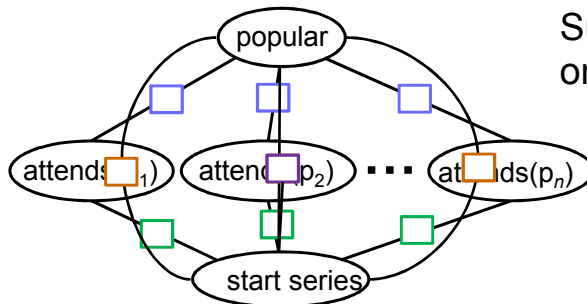
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## Variable Elimination



Sum out non-query variables  
one by one

$$\sum_{\text{att}(p_1)} \phi_1(\text{pop}, \text{att}(p_1)) \phi_2(\text{att}(p_1), \text{ser})$$

$\phi'(\text{pop}, \text{ser})$

Time is **linear** in  
number of invitees  $n$



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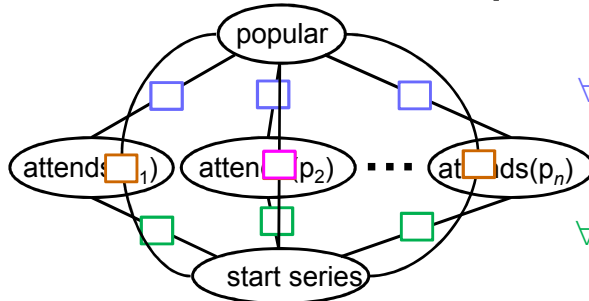


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## First-Order Variable Elimination

[Poole 2003; de Salvo Braz *et al.* 2005]



$$\forall X. \phi_1(\text{popular}, \text{attends}(X))$$

$$\forall X. \phi_2(\text{attends}(X), \text{series})$$



$$\forall X. \phi'(\text{popular}, \text{series})$$



$$\phi'(\text{popular}, \text{series})^n$$

Sum out all  $\text{attends}(X)$  variables at once

Time is **constant** in  $n$



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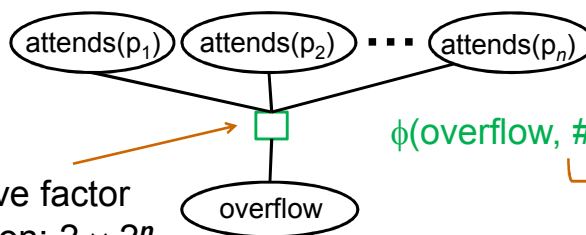


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## Symmetry Within Factors

[cf Zhang & Poole 1996; Gupta *et al.* 2007, Milch *et al.* 2008]



Size of naïve factor representation:  $2 \times 2^n$

$$\phi(\text{overflow}, \#_X[\text{attends}(X)])$$

**counting formula**

- Values of counting formula are **histograms** counting how many objects  $X$  yield each possible value of  $\text{attends}(X)$ 
  - Only  $n+1$  histograms, e.g., [50, 0], [49, 1], ..., [0, 50]
  - Factor size now  $2 \times (n+1)$ : **linear** in  $n$



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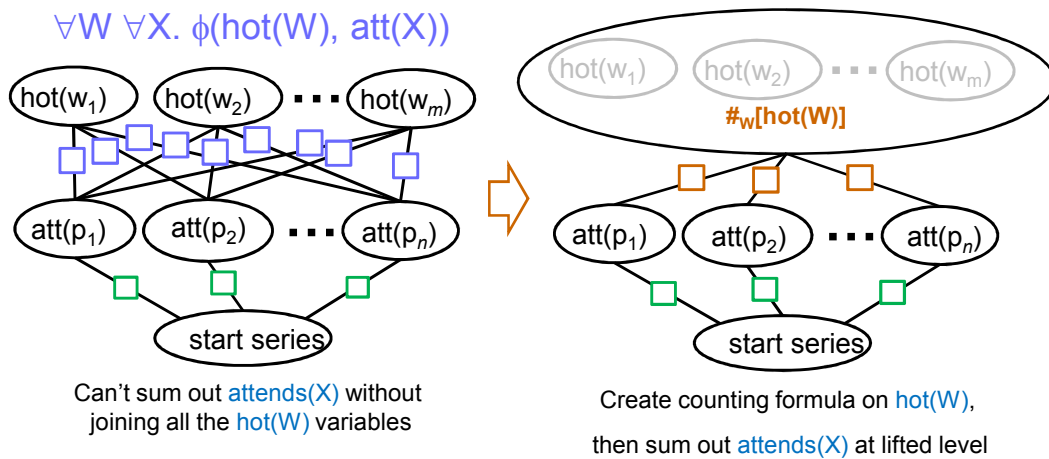
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## Example: Competing Workshops



Conversion to counting formulas creates new opportunities for lifted elimination

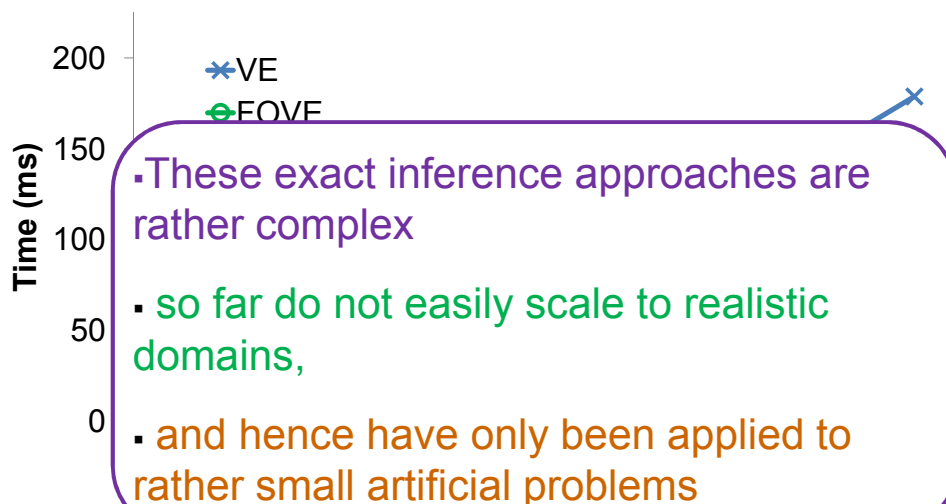


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## Results: Competing Workshops



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## How do you spend your spare time?



YouTube like media portals have changed the way users access media content in the Internet

Every day, millions of people visit social media sites such as Flickr, YouTube, and Jumpcut, among others, to share their photos and videos, ...

while others enjoy themselves by searching, watching, commenting, and rating the photos and videos; what your friends like will bear great significance for you.



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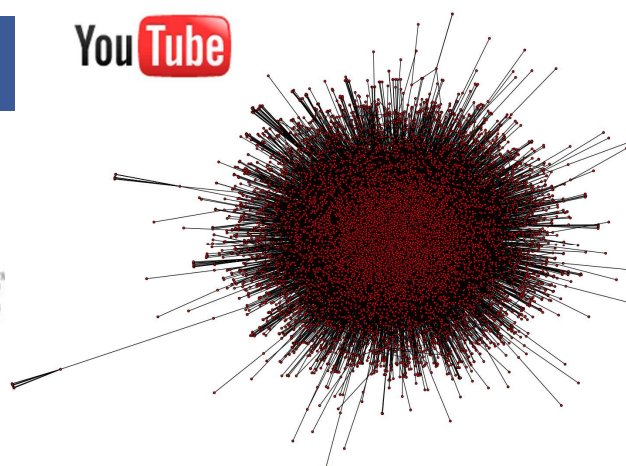
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## How do you efficiently broadcast information?



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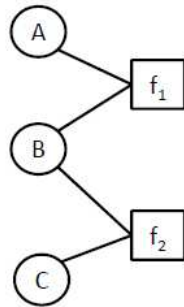
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## Content Distribution using Belief Propagation

[Bickson et al. 04]

- Approximate inference
- Compute the marginal  $P(x_i|X)$  for each  $x_i$  with local computations only
- Computer vision, combinatorial problems, SAT, NLP, ...



A	B	$f_1$
True	True	1.2
True	False	1.4
False	True	2.0
False	False	0.4

C	B	$f_2$
True	True	1.2
True	False	1.4
False	True	2.0
False	False	0.4

$$\mu_{X \rightarrow f}(x) = \prod_{h \in \text{nb}(X) \setminus \{f\}} \mu_{h \rightarrow X}(x)$$

$$\mu_{f \rightarrow X}(x) = \sum_{\neg\{x\}} \left( f(\mathbf{x}) \prod_{y \in \text{nb}(f) \setminus \{X\}} \mu_{y \rightarrow f}(y) \right)$$



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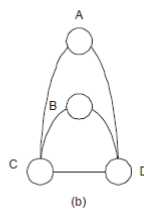
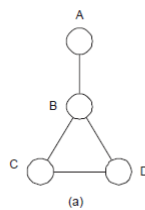


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## Content Distribution using Belief Propagation

[Bickson et al. WDAS04]



$\psi_{BB}(X_B)$	3 from A	1 from C	3 from C	3 from D
Uniform policy	1/4	1/4	1/4	1/4
Rarest part first	1/6	1/2	1/6	1/6

Table 2. Possible actions for the node B in the example graph shown in Figure 1

$\psi_{BD}(X_B, X_D)$	3 from A	2 from C	3 from C	3 from D
1 from B	1/8	1/8	1/8	1/8
2 from C	1/2	€	€	1/2
3 from C	1/2	€	€	1/2

Table 3. Example of the edge potentials for the edge BD for the graph shown in Figure 2. The matrix rows are then normalized.

**A lot of shared factors, so use  
lifted belief propagation**



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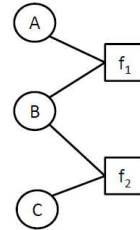
## Lifted Belief Propagation

[Singla, Domingos AAAI08, K, Ahmadi, Natarajan UAI09]

Counting shared factors can result in great efficiency gains for (loopy) belief propagation



Shared factors appear more often than you think in relevant real world problems



A	B	$f_1$
True	True	1.2
True	False	1.4
False	True	2.0
False	False	0.4

C	B	$f_2$
True	True	1.2
True	False	1.4
False	True	2.0
False	False	0.4

identical



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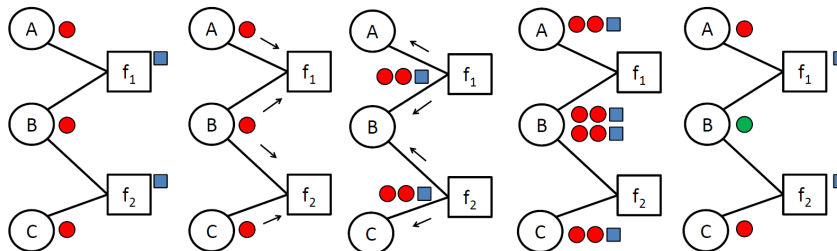
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## Step 1: Compression



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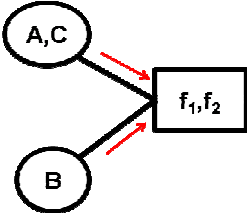


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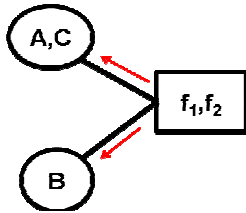
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## Step 2: Modified Belief Propagation



$$\mu_{\mathbf{x} \rightarrow \mathbf{f}}(x) = \mu_{\mathbf{f} \rightarrow \mathbf{x}}(x)^{c(\mathbf{f}, \mathbf{x})-1} \cdot \prod_{\mathbf{h} \in \text{nb}(\mathbf{x}) \setminus \{\mathbf{f}\}} \mu_{\mathbf{h} \rightarrow \mathbf{x}}(x)^{c(\mathbf{h}, \mathbf{x})}$$



$$\mu_{\mathbf{f} \rightarrow \mathbf{x}}(x) = \prod_{\mathbf{f} \in \text{nb}(\mathbf{x}_i)} \mu_{\mathbf{f} \rightarrow \mathbf{x}_i}(x_i)$$

$$b_i(x_i) = \prod_{\mathbf{f} \in \text{nb}(\mathbf{x}_i)} \mu_{\mathbf{f} \rightarrow \mathbf{x}_i}(x_i)^{c(\mathbf{f}, \mathbf{x})}$$



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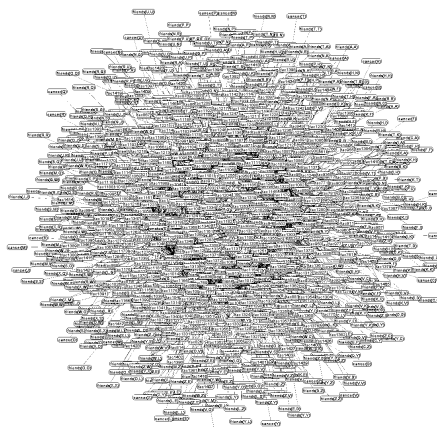
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## Social Network Analysis



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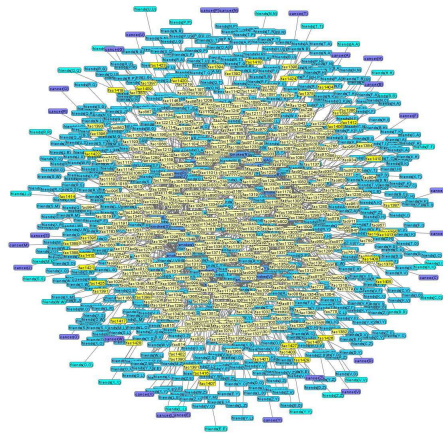


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## Social Network Analysis



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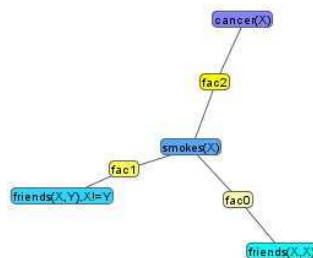
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## Social Network Analysis



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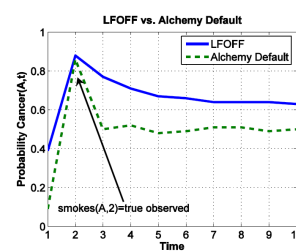
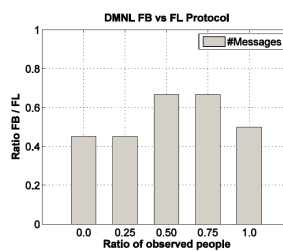
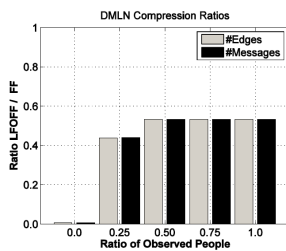
## Lifted First-order Factored Frontier

Apriori most people do not smoke  
 Apriori most people do not have cancer  
 Apriori most people are not friends  
 Smoking causes cancer  
 Friends have similar smoking habits  
 Most friends stay friends  
 Most smokers stay smokers

$\neg \text{Smokes}(x, 0)$   
 $\neg \text{Cancer}(x, 0)$   
 $\neg \text{Friends}(x, y, 0)$   
 $\text{Smokes}(x, t) \Rightarrow \text{Cancer}(x, t)$   
 $\text{Friends}(x, y, t) \Rightarrow (\text{Smokes}(x, t) \Leftrightarrow \text{Smokes}(y, t))$   
 $\text{Friends}(x, y, t) \Leftrightarrow \text{Friends}(x, y, \text{succ}(t))$   
 $\text{Smokes}(x, t) \Leftrightarrow \text{Smokes}(x, \text{succ}(t))$



20 people over 10 time steps. Max number of friends 5. Cancer never observed.  
Time step randomly selected.



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## Lower Bound on Model Count of CNF

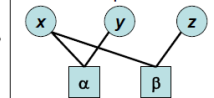
- BPCount [Kroc et al 08]
  - BP used to estimate marginals
  - Provable bound

e.g. SAT Problem:

$$\underbrace{(x \vee y)}_{\alpha} \wedge \underbrace{(\neg x \vee z)}_{\beta}$$



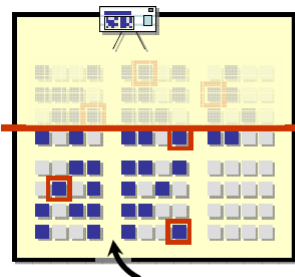
Factor Graph:



Idea:

- Identify a "balanced" row split or column split (roughly equal number of solutions on each side)
  - Use marginals for estimate
- Pick one side *at random*
- Count on that side recursively
- Multiply result by 2

[similar to decimation]



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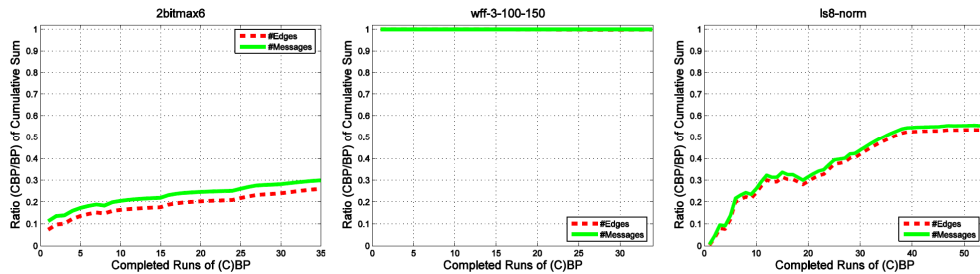


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## Model Counting



Satisfied by Lifted Message Passing?



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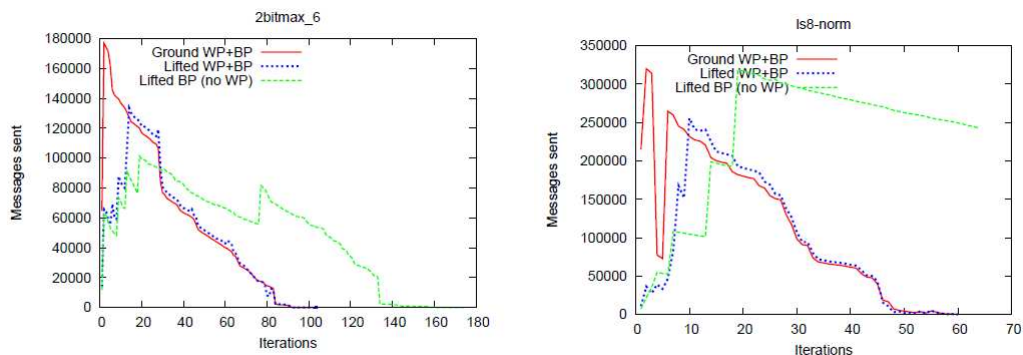


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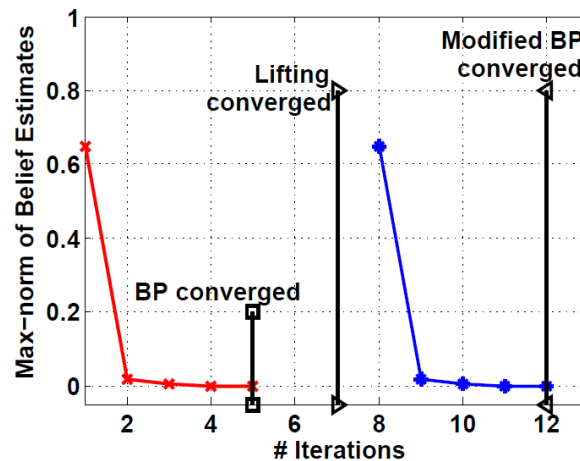
## Message Passing for Satisfiability

- Warning and survey propagation can also be lifted
- Enables lifted treatment of both prob. and det. knowledge



Gaussian Belief Propagation can also be lifted! Lifted Solvers for Systems of Linear Equations? Lifted Page Rank? Lifted HITS? Lifted Kalman Filter? ...

## Content Distribution (Gnutella): Lifted BP vs. BP



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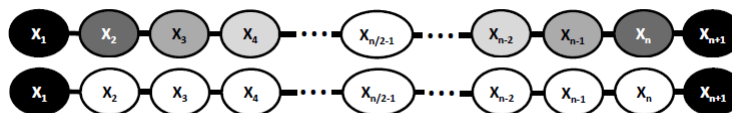


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## Message Errors to the Rescue!

- Ihler et al. 05: BP message errors decay along paths
- LBP may spuriously assume some nodes send and receive different messages and, hence, produce **pessimistic lifted network**



Make use of decaying message errors already at lifting time



K. Kersting  
Lifted Message Passing  
NeSy 2010 @ AAAI 2010,  
Atlanta, USA July 11, 2010



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## Informed Lifted Belief Propagation [El Massaoudi et al. AAAI10]

**Algorithm 1:** iLBP – informed Lifted BP. We use  $b_i(x_i)$  resp.  $m_i(x_i)$  to denote the unnormalized beliefs resp. messages of both variable node  $X_i$  and variable nodes covered by supernodes  $\mathcal{X}_i$ .

**Data:** A factor graph  $G$  with variable nodes  $X$  and factors  $f$ , Evidence  $E$

**Result:** Unnormalized marginals  $b_i(x_i)$  for all supernodes and, hence, for all variable nodes

```

1 Colorize  $X$  and  $f$  w.r.t.  $E$ ;
2  $\mathcal{G} \leftarrow$  one iteration CP;
3 Initialize messages for  $\mathcal{G}$ ;
4  $(b_i(x_i), m_i(x_i)) \leftarrow$  one iteration MBP on  $\mathcal{G}$ ;
5 Colorize all  $X_i$ s according to  $m_i(x_i)$ ;
6 while  $b_i(x_i)$ s have not converged do
7    $\mathcal{G}' \leftarrow$  one iteration CP (based on new colors);
8   Initialize novel supernodes using  $b_i(x_i)$  and  $m_i(x_i)$ ;
9    $(b_i(x_i), m_i(x_i)) \leftarrow$  one iteration of MBP on  $\mathcal{G}'$ ;
10  foreach supernode  $\mathcal{X}$  in  $\mathcal{G}$  do
11    if the  $m_i(x_i)$ s of the  $X_i$ s in  $\mathcal{X}$  differ then
12      | Colorize all  $X_i$  in  $\mathcal{X}$  according to  $m_i(x_i)$ 
13
14
15 Return  $b_i(x_i)$  for all supernodes
    
```

Iterate

Refine Lifting

Modified BP



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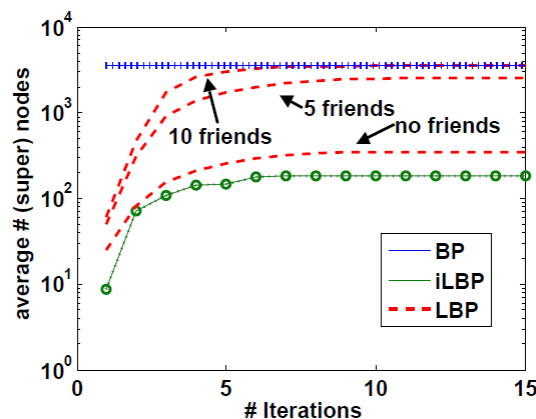
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## Social Networks



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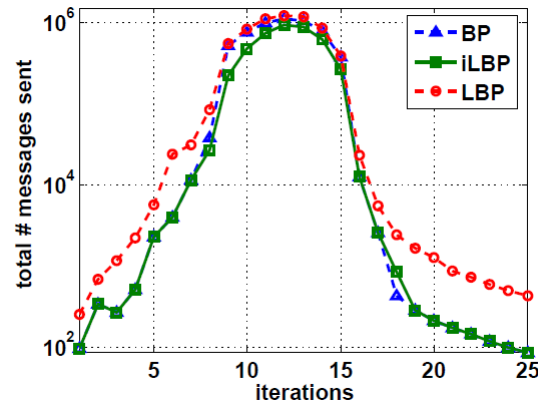


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## Lifted Content Distribution

- 1 file, Gnutella snapshot
  - 10876 nodes
  - 39994 edges
- iLBP 4.272.164 mess.
- < BP 5.761.952 mess.
- < LBP 6.381.516 mess.



- On a different network:
  - iLBP 1.972.662 < LBP 2.962.311 < BP 5.761.952



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## Conclusions

- $SRL \geq \text{Objects\&Relations} + \text{Probabilities} + \text{Machine Learning}$
- SRL works and covers the whole AI spectrum
  - Lifted Reasoning / SAT / Message Passing
  - Relational Machine Learning
  - Relational POMDPs [Sanner, K, AAAI10]
- Relational / Symbolic Neural Networks?
  - Deep Relational Networks?
  - Relational LPs?
  - Lifted Boltzmann Machines?
  - ...



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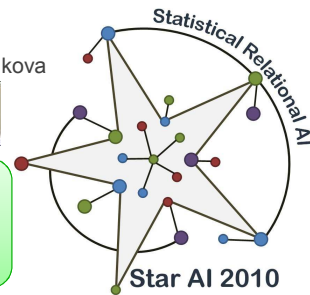
<http://www.biostat.wisc.edu/~natarasr/starai.html>

## StarAI@AAAI-10

Together with S. Russell, L. Kaelbling, A. Halevy, S. Natarajan, and L. Milhalkova



**Let's explore the minimal perturbations  
required for each of the AI areas to start  
using SR techniques**



- Planning: from PDDL to SRPDDL?
- MDPs: from 2-TBN models to DSRL models?
- Vision: from graphical models and scene grammars to SR generative models?
- NLP: from unification grammars to SR-unification grammars?
- KR: ontologies and event calculus to SR models?
- ...

**Thanks for your attention**



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